

Lens Terminology

The following definitions refer to the singlet lens diagram below. In the paraxial limit, however, any optical system can be reduced to the specification of the positions of the principal and focal points.

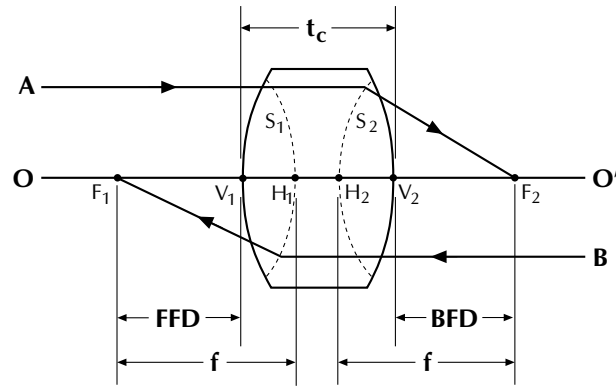


Figure 1. Principal Points, Focal Points, and Focal Lengths of a Generalized Optical System

Symbol	Description
O-O'	Optical Axis
F₁	Front Focal Point: Common focal point of all rays B parallel to optical axis and travelling from right to left.
V₁	Front Vertex: Intersection of the first surface with the optical axis.
S₁	First Principal Surface: A surface defined by the intersection of incoming rays B with the corresponding outgoing rays focusing at F1.
H₁	First Principal Point: Intersection of the first principal surface with the optical axis.
f	Effective Focal Length: The axial distance from the principal points to their respective focal points. This will be the same on both sides of the system provided the system begins and ends in a medium of the same index.
FFD	Front Focal Distance: The distance from the front vertex to the front focal point.
F₂	Back Focal Point: Common focal point of all rays A parallel to optical axis and traveling from left to right.
V₂	Back Vertex: Intersection of the last surface with the optical axis.
S₂	Second Principal Surface: A surface defined by the intersection of rays A with the outgoing rays focusing at F2.
H₂	Second Principal Point: Intersection of the second principal surface with the optical axis.
BFD	Back Focal Distance: The distance from the back vertex to the back focal point.

Focal Length Formulas for Singlet Lenses

Type	Orientation	Focal Length f	Back Focal Distance BFD	Front Focal Distance FFD
General	$R_1 = R_1$ $R_2 = R_2$	$\left[(n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{t_c (n-1)^2}{nR_1R_2} \right]^{-1}$	$f \cdot \left[1 - \frac{t_c (n-1)}{nR_1} \right]$	$f \cdot \left[1 + \frac{t_c (n-1)}{nR_2} \right]$
Plano-Convex	$R_1 = R$ $R_2 = \infty$	$\frac{R}{n-1}$	$f \cdot \left[1 - \frac{t_c (n-1)}{nR} \right]$	f
Plano-Concave	$R_1 = -R$ $R_2 = \infty$	$\frac{-R}{n-1}$	$f \cdot \left[1 + \frac{t_c (n-1)}{nR} \right]$	f
Bi-Convex	$R_1 = R$ $R_2 = -R$	$\left[\frac{2(n-1)}{R} - \frac{t_c (n-1)^2}{nR^2} \right]^{-1}$	$f \cdot \left[1 - \frac{t_c (n-1)}{nR} \right]$	$f \cdot \left[1 - \frac{t_c (n-1)}{nR} \right]$
Bi-Concave	$R_1 = -R$ $R_2 = R$	$-\left[\frac{2(n-1)}{R} - \frac{t_c (n-1)^2}{nR^2} \right]^{-1}$	$f \cdot \left[1 + \frac{t_c (n-1)}{nR} \right]$	$f \cdot \left[1 + \frac{t_c (n-1)}{nR} \right]$

Singlet Lenses

In the table, we tabulate the focal length, back focal distance, and front focal distance for a general singlet lens and the commonly available singlet lenses plano-convex, plano-concave, bi-convex (actually equiconvex), and bi-concave (actually equiconcave). The sign conventions used are as follows:

- a. R is positive when the surface is convex when encountered moving from left to right along the optical axis
- b. R is negative when the surface is concave when encountered moving from left to right along the optical axis
- c. BFD is positive when F_2 is to the right of V_2 . BFD is negative when F_2 is to the left of V_2
- d. FFD is positive when F_1 is to the left of V_1
- e. FFD is negative when F_1 is to the right of V_1
- f. The first principal point H_1 is to the right of F_1 for positive f , and to the left of F_1 for negative f
- g. The second principal point H_2 is to the left of F_2 for positive f , and to the right of F_2 for negative f

Note that the formulas for FFD and BFD depend on the lens orientation when the lens is asymmetrical. The formula for focal length (f) does not depend on orientation. Finally, when R is used without a subscript in the formulas below, it is positive.

Using the formulas in the table, you can plot out the principal points. Using the formulas for FFD and BFD, one can find the positions of the front and back focal

points with respect to the front and back vertices. Using the value and sign of f , one can find the first and second principal points.

Spherical Aberration and Spot Size

High quality singlet lenses are of particular interest in laser focusing and beam handling applications because of their low cost, high damage threshold, and the availability of standard parts. CVI offers a large selection of BK7 and UV grade fused silica singlets that can satisfy most laser focusing requirements.

Can singlet lenses provide diffraction limited focusing of collimated beams? Examine the Figure 2.

This figure represents 25 light rays traced near the focal plane of an $f=+100\text{mm}$ plano-convex lens of index $n=1.515$. The radius of curvature of the convex surface is $R = 51.5\text{mm}$ and its center thickness is $t_c = 6\text{mm}$. The rays start out parallel to the optical axis, and were spaced equally in a region $\pm 16\text{mm}$ above and below it. Therefore the lens is operating at a relatively fast $f/\#$:

$$f/\# = \frac{f}{d} = \frac{100\text{mm}}{32\text{mm}} = 3.125$$

The back focal distance of the lens is:

$$\begin{aligned} \text{BFD} &= f \cdot \left[1 - \frac{t_c (n-1)}{nR} \right] \\ &= f - \frac{t_c}{n} = 96.04\text{mm} \end{aligned}$$

Clearly, not all rays come to focus in the paraxial focal plane, marked PP in the figure. Only rays very close to the optical axis focus there. Rays that start out *farther* (marginal offset) from the optical axis

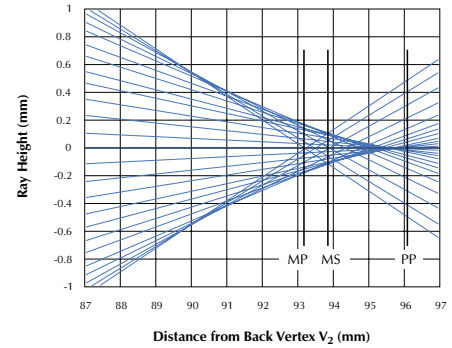


Figure 2. Ray trace near the focus of a 100mm lens.

come to a focus *closer* to the lens. The marginal rays, which started out 16mm from the optical axis, focus in plane MP. There is a minimum beam diameter d_{MS} which occurs at intermediate plane MS. d_{MS} can be calculated from Third Order aberration theory. The result is:

$$d_{MS} = f \cdot \Delta\theta_{\text{blur}}$$

where $\Delta\theta_{\text{blur}}$ is the angular blur of the lens, given by:

$$(d_0/f)^3 \cdot \left[\frac{n^2 - (2n+1)K + (n+2)K^2/n}{32(n-1)^2} \right]$$

d_0 is the initial beam diameter and K is the shape factor of the lens:

$$K = \frac{R_2}{R_2 - R_1}$$

The full angular blur is plotted in Figure 3 for singlet lenses as a function of shape factor for various indices of refraction. For $n=1.5$, the blur is a minimum for $K=6/7$. This describes a bi-convex lens with the flatter side $R_2 = -6R_1$. Such a lens is called a *best form singlet lens*, and will focus to a spot approximately 8% smaller than a plano-convex lens.

continued

In our example of a plano-convex lens of $f=+100\text{mm}$ oriented with plano side toward the focus, $K=1$. Using $d_0 = 32\text{mm}$, we compute $d_{MS} = 0.226\text{mm}$ due to the transverse spherical aberration component of Third Order theory. The wavelength of light has not entered the calculation; this result is a pure geometric optics calculation.

The diffraction limited focusing of a uniform 32mm diameter beam by a 100mm lens can be estimated by the diameter of the first minimum of the Airy diffraction pattern in the focal plane. If we take $\lambda=632.8\text{nm}$, we obtain:

$$d_{\text{diff}} = f \cdot \Delta\theta_{\text{diff}} = 2.44 (\lambda f / d_0) = 4.8 \mu\text{m}$$

Geometric optics theory predicts a beam diameter, $d_{MS} = 266\mu\text{m}$.

Focal length of lens (f): 96.04mm
 Incident Beam Ø: 32mm collimated
 Spot size results from Third Order theory calculation of $266\mu\text{m}$ and from diffraction limit calculation of $4.8\mu\text{m}$. This lens will exhibit a theoretical spot size approximately 47 times the diffraction limit. In other words, spherical aberration contributes an intrinsic blur, $\Delta\theta_{\text{blur}}$, which limits the angular resolution of the lens. At this point

$$\Delta\theta_{\text{blur}} > \Delta\theta_{\text{diff}}$$

we may say that a singlet, if fabricated with high precision, approaches diffraction limited performance.

Singlet or Multiple Element Lens?

Here is a procedure to determine whether a singlet or multiple element lens will be needed to achieve the required spot size.

1. Determine the spot size d_{spot} needed for the experiment.

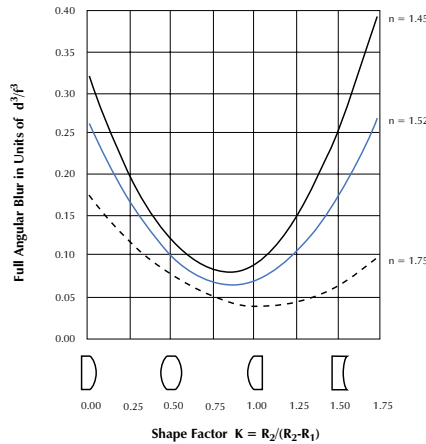


Figure 3. Full angular blur as a function of shape factor.

2. Calculate the required focal length from the divergence properties of the beam.

For a TEM₀₀ gaussian beam, use:

$$f = \frac{d_{\text{spot}}}{\Delta\theta_{\text{full}}} \approx \frac{\pi W_{\text{spot}} W_0}{\lambda}$$

where w_0 and w_{spot} are the initial and desired beam waists. This formula is an excellent approximation when the distance from the initial waist w_0 to the front focal plane is much smaller than the initial confocal parameter $z_0 = \pi w_0^2 / \lambda$.

For a plane wave truncated by an aperture d_0 use:

$$f = \frac{d_{\text{spot}}}{\Delta\theta_{\text{full}}}$$

For a non-diffraction limited laser beam, use:

$$f = \frac{d_{\text{spot}}}{\Delta\theta_{\text{diff}}} \approx \frac{d_{\text{spot}} d_0}{2.44 \lambda}$$

where $\Delta\theta_{\text{full}}$ is the measured or manufacturer specified full angle divergence.

3. Calculate the blurred focal spot due to spherical aberration, using the shape factor, index of refraction, and initial beam diameter:

$$d_{MS} = f \Delta\theta_{\text{blur}} = (d_0^3 / f^2) \cdot \left[\frac{n^2 - (2n + 1)K + (n + 2)K^2 / n}{32(n - 1)^2} \right]$$

For a properly oriented plano-convex lens of index $n=1.5$, the bracketed factor is 0.073.

If the blurred focal spot size is greater than or equal to the desired spot size, a multi-element lens is required to achieve the desired spot size. This lens selection rule is based on blur as a result of spherical aberration compared with blur due to diffraction. It can be formulated as a limiting $f/\#$. For plane wave focusing:

$$2.44 \frac{\lambda}{d_0} = \left(\frac{d_0}{f} \right)^3 \left[\frac{n^2 - (2n + 1)K + (n + 2)K^2 / n}{32(n - 1)^2} \right] = \alpha \cdot \left(\frac{d_0}{f} \right)^3$$

Using

$$f/\# = \frac{f}{d_0}$$

we obtain the condition

$$f/\# > \left(\frac{\alpha f}{2.44 \lambda} \right)^{1/4}$$

To achieve diffraction limited focusing, the $f/\#$ has to be greater, or the speed of the single element lens "slower" than this expression containing the focal length. As an example, see Figure 4.

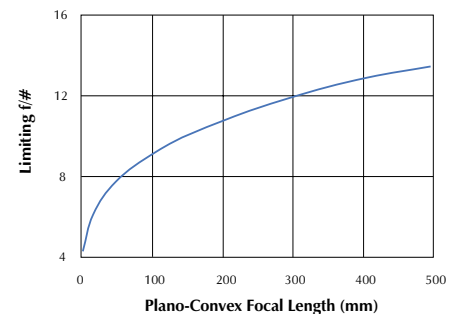


Figure 4. Limiting $f/\#$ for diffraction limited focusing for a plano-convex fused silica lens at $\lambda = 514.5\text{nm}$

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Surface Quality

Scratch-Dig	Relative Cost	Applications
60-40	Very Low	Used in low power laser and imaging applications where scattered light is not as critical as cost.
40-20	Low	For laser and imaging applications with focused beams that tolerate little scattered light.
20-10	Moderate	For laser and imaging applications with focused beams where minimizing scattered light is more critical. Best quality offered by typical catalog houses.
20-10 CVI LQ	Moderate	CVI Laser Quality level of inspection. For laser and imaging applications with focused beams where minimizing scattered light is more critical. More details on page 430.
10-5 CVI LQ	Moderately High	CVI Laser Quality level of inspection. Required for high damage threshold in high laser energy applications. Best performance for laser material processing applications and laser cavity optics. More details on page 430.

Surface Figure/Transmitted Wavefront Distortion

Surface Figure	Relative Cost	Applications
$\lambda/2$	Very Low	Cost optimization solution. Also used with very fast or short radius singlets.
$\lambda/4$	Low	For general laser and imaging applications where wavefront performance is balanced with cost.
$\lambda/8$	Moderate	For laser and imaging applications with low wavefront distortion requirement, especially in multi-element systems. Best quality offered by typical catalog houses.
$\lambda/10$	Moderately High	CVI signature wavefront quality level. Required for best performance in ultra-violet and performance critical applications.

CVI maintains a certified ISO9001:2000 quality management system through a 100% outgoing inspection. We have invested in Zygo-DPI interferometers for all of our facilities. Our optics are measured against best-in-class references. We use the latest in computer generated holograms to test our cylindrical lenses.



CVI's 33-15 and 33-10SET Lens Holders ▶ 394



CVI's Model 3100 Lens Centering Cell ▶ 393